THE EXCHANGE OF HEAT BETWEEN A LIQUID SPHEROID AND THE AMBIENT MEDIUM IN THE PRESENCE OF LEIDENFROST PHENOMENON

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A two-dimensional problem in the hydrodynamics of a vapor layer and the process of heat transfer under conditions of the Leidenfrost phenomenon is examined in the exchange of heat between a liquid spheroid and the ambient medium.

A review and the contemporary state of various questions dealing with the experimental and theoretical study of the Leidenfrost phenomenon are covered in [1-5], where an extensive bibliography can be found. The influence of individual factors such as drop dimension, roughness of the heated surface, the degree of initial underheating, etc., on the Leidenfrost temperature T_L has been examined in rather great detail. As a rule, however, little consideration has been given to the effect on T_L from the exchange of heat between a liquid spheroid and the ambient medium. Nevertheless, the very first experimental studies [6] have demonstrated qualitatively the nonuniformity of the temperature field in the spheroid, which, all other factors aside, is apparently also associated with the process of heat exchange which takes place between the outer surface of the spheroid and the ambient medium. It is the purpose of this paper to take approximate account of the exchange of heat between the liquid spheroid and the ambient medium, as well as of the two-dimensionality of the vapor flow beneath the spheroid.

Let us consider the case of rather large spheroids in which the liquid drop is similar in shape to a disk of thickness H and radius R. As our basic and frequently utilized assumptions we will take those formulated in [4]. We will additionally assume that the exchange of heat between the spheroid and the ambient medium at the upper and side surfaces of the spheroid satisfies boundary conditions of the third kind. We will also neglect the convection of the liquid within the spheroid, which is a consequence of the relationship between the density of the liquid and the temperature, the Marangoni effect, etc.

Let us introduce a cylindrical system of coordinates, setting the origin of the Z axis at the heating surface. Then, within the framework of quasisteady approximation, the hydrodynamics and heat transfer of the "spheroid-vapor layer" system considered will be described by the following system of equations:

$$\frac{\partial P}{\partial r} = \mu \frac{\partial^2 u_r}{\partial z^2}; \quad \frac{\partial P}{\partial z} = 0; \quad r\frac{\partial u_z}{\partial z} + \frac{\partial (r u_r)}{\partial r} = 0; \tag{1}$$

$$u_r \frac{\partial T}{\partial r} + u_z \frac{\partial T}{\partial z} = a \frac{\partial^2 T}{\partial z^2}; \tag{2}$$

$$\frac{\partial^2 T_\ell}{\partial r^2} + \frac{1}{r} \frac{\partial T_\ell}{\partial r} + \frac{\partial^2 T_\ell}{\partial z^2} = 0 \tag{3}$$

with boundary conditions

$$z = 0; \quad u_r = u_z = 0; \quad T = T_w; \tag{4}$$

$$z = h; \quad u_r = 0; \quad u_z = -V; \quad T = T_\ell = T_s; \tag{5}$$

$$-\lambda \frac{\partial T}{\partial z} = -\lambda_\ell \frac{\partial T_\ell}{\partial z} + \rho V L; \tag{6}$$

$$z = H + h; \quad -\lambda_\ell \frac{\partial T_\ell}{\partial z} = \alpha_1 (T_\ell - T_c); \tag{7}$$

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$$r=0;\quad T_\ell \neq \infty;\quad T=F(z);\quad \frac{dP}{dr}=0; \tag{8}$$

$$r=R;\quad P=P_c;\quad -\lambda_\ell \frac{\partial T_\ell}{\partial r}=\alpha_2 (T_\ell - T_c). \tag{9}$$

Here boundary condition (9) for liquid temperature is taken with the additional assumption that the exchange of heat at the side surface of the drop, spheroidal in shape, generally speaking, in first approximation can be represented as the exchange of heat at the cylindrical side surface of a disk with a constant coefficient of heat transfer α_2.

Speaking of boundary condition (8) for vapor temperature, it should be noted that in the real problem it is difficult to specify a law governing the distribution of the temperature in the vapor layer at the symmetry axis. Therefore, it is proposed to interpolate by an exponential series with respect to the transverse coordinate this function F(z), limiting ourselves in each specific problem to a finite number of series terms. In this case, the expansion factors must be sought from among additional physical concepts.

Let us note that the velocity V of the vapor at the boundary of phase separation and the thickness of the vapor layer are not known in advance and must be determined during the course of solving the formulated problem. With consideration of the aforesaid, we must additionally take into consideration the condition of liquid-drop equilibrium on the vapor layer

$$\rho_\ell g \pi R^2 H = 2\pi \int_0^R r(P-P_c)\,dr. \tag{10}$$

This last condition is written with consideration of the assumption to the effect that the reaction of the removed vapor is negligibly small in comparison with the drop force of gravity. It should be noted that Eq. (10) is approximate in nature and is used here in conjunction with the concept of a liquid spheroid in the form of a rigid disk in the presence of significant underheating. Resorting to boundary conditions (7)-(9) for the temperature of the liquid, we can write the solution of Eq. (3) with the familiar methods of [7] in the form

$$T_\ell = T_c + \sum_{n=1}^{\infty} C_n \left\{ \exp\left(-\frac{\varepsilon_n z}{R}\right) - \frac{(\mathrm{Bi}_1-\varepsilon_n)}{(\mathrm{Bi}_1+\varepsilon_n)} \exp\left(\frac{\varepsilon_n (z-2(H+h))}{R}\right)\right\} J_0\left(\frac{\varepsilon_n r}{R}\right), \tag{11}$$

where C_n are the expansion factors as yet unknown; ε_n are the roots of the characteristic equation $\mathrm{Bi}_2 J_0(\varepsilon) = \varepsilon J_1(\varepsilon)$. Using boundary condition (5) for the liquid temperature, with consideration of the orthogonality of the basis functions, from (11) we find that

$$C_n = \frac{2(T_s - T_c) J_1(\varepsilon_n)}{\varepsilon_n \beta_n [J_0^2(\varepsilon_n) + J_1^2(\varepsilon_n)]};$$
$$\beta_n = \exp\left(-\frac{\varepsilon_n h}{R}\right) - \frac{(\mathrm{Bi}_1-\varepsilon_n)}{(\mathrm{Bi}_1+\varepsilon_n)} \exp\left(-\frac{\varepsilon_n (2H+h)}{R}\right). \tag{12}$$

Let us note that for the final determination of C_n we must know the thickness of the vapor layer.

The temperature distribution in the vapor layer is sought in the form of a series over the transverse coordinate

$$T(r,\ z) = \sum_{k=0}^{\infty} z^k T_k(r), \tag{13}$$

where $T_k(r)$ are functions as yet unknown. Having determined the vapor-velocity vector projections from (1), with consideration of boundary conditions (4) and (5) for u_r and condition (4) for u_z, and having substituted these together with (13) into (2), after regrouping of terms we obtain the following system of differential equations:

$$\frac{1}{2\mu}\left\{\frac{dT_{k-2}}{dr}-h\frac{dT_{k-1}}{dr}\right\}\frac{dP}{dr}-a(k+2)(k+1)T_{k+2}+$$
$$+\frac{1}{2\mu r}\left\{\frac{(k-1)hT_{k-1}}{2}-\frac{(k-2)T_{k-2}}{3}\right\}\frac{d}{dr}\left(r\frac{dP}{dr}\right)=0, \tag{14}$$
$$k=0,\ 1,\ \ldots;\quad T_{-1}\equiv 0;\quad T_{-2}\equiv 0.$$

A unique feature of this derived system of equations is the fact that if in it we limit ourselves to a finite number of, for example, the first N, equations, we will always come to a subsystem containing N + 3 unknown functions: P, T_0, T_1, ..., T_{N+1}. The first two missing equations follow from boundary conditions (4) and (5) for the vapor temperature and with consideration of (13) have the form

$$T_0=T_w;\quad T_s=\sum_{k=0}^{\infty}h^kT_k(r). \tag{15}$$

The last missing equation follows from (6) with consideration of (11) and (13) as well as from the solution of system (1) with boundary condition (5) for u_z and can be written in the form

$$-\lambda\sum_{k=0}^{N+1}kh^{k-1}T_k(r)=\frac{\lambda_\ell}{R}\sum_{n=1}^{\infty}C_n\varepsilon_n\gamma_nJ_0\left(\frac{\varepsilon_nr}{R}\right)-$$
$$-\frac{\rho Lh^3}{12\mu r}\frac{d}{dr}\left(r\frac{dP}{dr}\right); \tag{16}$$
$$\gamma_n=\exp\left(-\frac{\varepsilon_nh}{R}\right)+\frac{(\mathrm{Bi}_1-\varepsilon_n)}{(\mathrm{Bi}_1+\varepsilon_n)}\exp\left(-\frac{\varepsilon_n(2H+h)}{R}\right).$$

Thus the first N equations from (14) in combination with (15) and (16) form a closed system of nonlinear differential equations for determination within the scope of this approximation of the pressures and temperatures within the vapor layer, where consideration is given to boundary conditions (8) and (9). The last unknown parameter h is determined after finding P as a solution of Eq. (10).

Let us now consider the simpler special case of constructing an approximate solution of this problem, when in (14) we can limit ourselves to the single first equation (N = 1). It is not difficult to see that such a special case corresponds to the frequently employed linear interpolation of the temperature in the vapor layer as a function of the lateral z coordinate. Then from the system of equations (14)-(16) we find

$$P=P_c+\frac{12\mu}{\rho Lh^3}\left\{\frac{\lambda(R^2-r^2)(T_w-T_s)}{4h}+\right.$$
$$\left.+\lambda_\ell R\sum_{n=1}^{\infty}\frac{C_n\gamma_n}{\varepsilon_n}\left(J_0(\varepsilon_n)-J_0\left(\frac{\varepsilon_nr}{R}\right)\right)\right\}. \tag{17}$$

The projections of the vapor velocity vector onto the coordinate axes for the special case under consideration from the solution of system of equations (1) with consideration of (17) have the form

$$u_r=\frac{6(z^2-hz)}{\rho Lh^3}\left\{-\frac{\lambda(T_w-T_s)r}{2h}+\lambda_\ell\sum_{n=1}^{\infty}C_n\gamma_nJ_1\left(\frac{\varepsilon_nr}{R}\right)\right\},$$
$$u_z=\frac{(2z^3-3hz^2)}{\rho Lh^3}\left\{\frac{\lambda(T_w-T_s)}{h}-\frac{\lambda_\ell}{R}\sum_{n=1}^{\infty}C_n\gamma_n\varepsilon_nJ_0\left(\frac{\varepsilon_nr}{R}\right)\right\}. \tag{18}$$

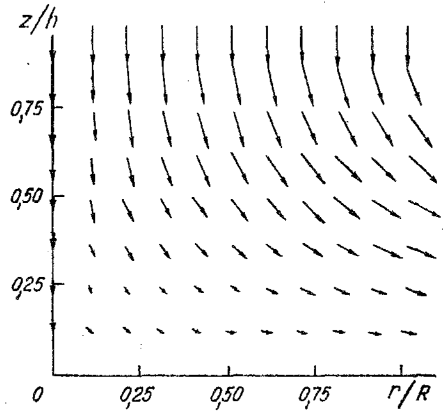

Fig. 1. Dimensionless velocity-vector field in a vapor layer beneath a liquid spheroid.

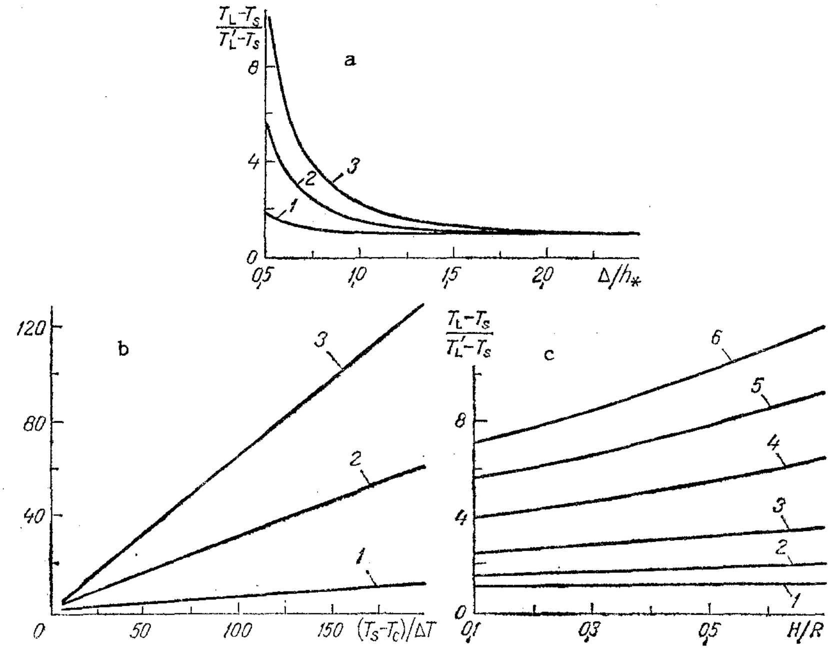

Fig. 2. The relationship between $(T_L - T_s)/(T_L' - T_s)$ and Δ/h_* (a), $(T_s - T_c)/\Delta T$ (b) and H/R (c): $\lambda_\ell/\lambda = 30$, $h_*/R = 0.01$. a) H/R = 0.7, $(T_s - T_c)/\Delta T = 15$: 1) $Bi_1 = Bi_2 = 0.1$; 2) 1.0; 3) 10.0. b) H/R = 0.7, $\Delta/h_* = 0.5$: 1) $Bi_1 = Bi_2 = 0.1$; 2) 1.0; 3) 10.0. c) $\Delta/h_* = 0.5$, $Bi_1 = Bi_2 = 0.1$: 1) $(T_s - T_c)/\Delta T = 5$; 2) 20; 3) 50; 4) 100; 5) 150; 6) 200.

As an illustration Fig. 1 shows the field of the dimensionless velocity vector in the vapor layer beneath the liquid spheroid, constructed with consideration of (18). In this case, as the scale of the velocity we chose the maximum value of u_r attained when z = h/2 and r = R. The calculations were carried out with the following basic system parameters: H/R = 0.3, $Bi_1 = Bi_2 = 0.1$, h/R = 0.01054, $\lambda_\ell/\lambda = 30$, $(T_s - T_c)/(T_w - T_s) = 0.3$. In view of the smallness of the values for the dimensionless lateral vapor velocity component, for convenience of construction this component of the velocity vector was increased by a factor of R/h, which in the example under consideration amounted to 94.876.

To determine the last unknown parameter of the problem, we will substitute (17) into (10). Then, after appropriate transformations with consideration of (12), we obtain the following equation for the determination of the vapor-layer thickness:

$$\frac{T_w - T_s}{\Delta T} = 8\left(\frac{h}{h_*}\right)^4 - 16\mathrm{Bi}_2 \frac{\lambda_\ell}{\lambda} \frac{h_*}{R} \frac{T_s - T_c}{\Delta T} \left(\frac{h}{h_*}\right) \sum_{n=1}^{\infty} \frac{\gamma_n [\varepsilon_n^2 - 2\mathrm{Bi}_2]}{\beta_n \varepsilon_n^3 [\varepsilon_n^2 + \mathrm{Bi}_2^2]}. \tag{19}$$

Here

$$h_* = \sqrt{2}R\left(\frac{3\mu\lambda T_s}{\rho^2 L^2}\right)^{\frac{1}{4}};\ \Delta T = \frac{\rho_\ell g H T_s}{\rho L},$$

where certain characteristic values corresponding to the thickness of the vapor layer and the temperature difference have been taken as the scale magnitudes.

Let us examine the question of the effect exerted by consideration of the liquid-spheroid heat exchange with the ambient medium on the calculated values of the Leidenfrost temperature. For T_L we traditionally assume a temperature of the heated surface such that the spheroid at its base is in contact with the apices of its roughness. Assuming in this case that

$$T_w = T_L;\quad h = \Delta, \tag{20}$$

from (19) we obtain the expression for calculation of the Leidenfrost temperature with consideration of the exchange of heat between the liquid of the spheroid and the ambient medium.

After we have compared the expression that we obtained in this manner with the familiar results, we note the following. If we neglect the exchange of heat between the spheroid and ambient medium (which is the case when $\mathrm{Bi}_1 = \mathrm{Bi}_2 = 0$), the obtained relationship leads directly to the familiar asymptote

$$\frac{T_L' - T_s}{\Delta T} = 8\left(\frac{\Delta}{h_*}\right)^4, \tag{21}$$

which is in good agreement for the case of rather large Δ/h_* with the results in [5].

Figure 2a shows the calculated values of the Leidenfrost temperature in dimensionless relative coordinates with consideration of the exchange of heat between the spheroid and the ambient medium as a function of the roughness of the heated surface. As follows from these data, the presence of spheroid heat exchange for comparatively small values of Δ/h_* leads to a noticeable increase in the Leidenfrost temperature. And, conversely, when the relative roughness exhibits rather large values, we find that the effect of taking into consideration the exchange of heat from the spheroid as it regards the anticipated increase in the Leidenfrost temperature is rather weakly expressed.

The effect of the temperature of the ambient medium and H/R on $(T_L - T_S)/(T_L' - T_S)$ for certain characteristic values of the remaining parameters may be illustrated as an example by the relationships in Fig. 2b, c.

As we were constructing these relationships we kept in mind (19), as well as (20), for the case in which $\mathrm{Bi}_1 = \mathrm{Bi}_2$, and also the well-known relationship (21).

The results considered here indicate the need in a number of cases of the Leidenfrost phenomenon to account for the exchange of heat of the liquid spheroid and the ambient medium. In this case, consideration of the spheroid heat exchange leads to an increase in the calculated values of the Leidenfrost temperature relative to the values which might be derived if it were not taken into consideration. We can expect that in the case of a more realistic model of the process the effect of allowing for the exchange of heat of the spheroid as far as the calculated values of the Leidenfrost temperatures are concerned will be more pronounced. Apparently, this may be associated with the presence of the convection mechanism of heat transfer within the spheroid, which was not taken into consideration in the model examined here.

The formulas presented here, and which correspond to the simplest case in which in (14) it is acceptable to limit ourselves to only a single first equation (N = 1), naturally are approximate in nature. However, with accuracy to within all of the above assumptions, they do allow us to obtain preliminary quantitative estimates of the extent to which the heat exchange of the liquid spheroid with the ambient medium affects the hydrodynamics and heat-transfer process in the vapor layer. In those cases in which accuracy must be raised, we have to limit ourselves to a larger number (N ≥ 2) of first equations in system (14).

NOTATION

r, z, cylindrical coordinates; u_r, u_z, projections of the vapor velocity vector onto the coordinate axes; T, T_ℓ, temperatures in the vapor layer and in the liquid spheroid; μ, a, coefficients of vapor viscosity and thermal conductivity; P, pressure within the vapor layer; V, velocity of the vapor at the boundary of phase separation; λ and λ_ℓ, coefficients of vapor and liquid thermal conductivity; T_s, T_w, T_c, saturation temperatures for the liquid, the heated surface, and the ambient medium; P_c, pressure in the ambient medium; ρ, ρ_ℓ, density of the vapor and of the liquid; L, specific heat of vaporization; α_1, α_2, heat-transfer coefficients at the upper and side surfaces of the spheroid; H, R, thickness and radius of the spheroid; h, thickness of the vapor layer; g, gravitational acceleration; J_0, J_1, Bessel functions of the first kind, of zeroth and first order; h_*, ΔT, characteristic values of the vapor-layer thickness and the temperature difference; Δ, roughness magnitude of the heated surface; T_L, T_L', Leidenfrost temperature for the cases in which consideration is given to and not given to the exchange of heat between the liquid spheroid and the ambient medium. Criteria: $Bi_1 = \alpha_1 R/\lambda_\ell$; $Bi_2 = \alpha_2 R/\lambda_\ell$.

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THE SPREADING OF A MICROSTRUCTURAL FLUID OVER A SOLID SURFACE

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The kinetics involved in the spreading of a drop of microstructural fluid over a horizontal solid surface is investigated theoretically. A method is proposed for the measurement of material constants of the fluid, characterizing its micropolarity.

1. Specialists in the field of hydrodynamics and physical chemistry have recently paid particular attention to problems of fluid spreading and displacement of the contact line between fluid 1, fluid 2, and a solid surface [1-9]. Considerable progress has been achieved at this time in this area, but at the same time all of the attempts theoretically to analyze the problems of spreading encounter two fundamental difficulties.

The first difficulty involves the shifting of the contact line, since the Navier–Stokes equations for the boundary conditions of adhesion lead to an impermissible singularity in the force on this line [1, 2]. There exists a means of eliminating this singularity of force by means of utilizing the condition of slippage or shear in the region of the contact line, e.g., the Maxwell condition at which the magnitude of the shear is proportional to the local velocity gradient [3, 4].

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